# Math 174E Lecture 1–3: Core Definitions and Concepts

**Definition 1** ((1.1) Financial Asset and Security)**.** *A* ***financial asset*** *is an asset whose value is derived from a contractual right or ownership claim. A* ***financial security*** *is a tradeable financial asset.*

**Definition 2** ((1.2) Corporation and Stock)**.** *A* ***corporation*** *is a legally defined artificial entity, separate from its owners. Ownership is divided into shares of* ***stock****. The total collection of shares is called the* ***equity*** *of the corporation. Stockholders (shareholders, equity holders) are partial owners. One share provides fractional ownership, voting rights, and potential dividend payouts. Shares may be* ***common*** *or* ***preferred****, and corporations may be* ***private*** *or* ***public****.*

**Definition 3** ((1.3) Stock Market)**.** *A* ***stock market*** *is an organized venue where shares of corporations are publicly traded. It provides liquidity and price discovery via supply and demand.*

**Proposition 1** ((1.A) Types of Markets and Offerings)**.** *Primary markets handle new issues, such as an initial public offering (IPO), where firms sell shares to raise capital. Secondary markets allow trading of existing securities among investors. IPOs determine the* ***free float****, the fraction of shares publicly traded.*

**Proposition 2** ((1.B) U.S. Stock Market Structure and Regulation)**.** *Trading is regulated by the* ***SEC****, created after the 1929 crash to prevent fraud and manipulation. It mandates corporate disclosures and monitors fair trading. The two dominant exchanges are the* ***NYSE*** *and* ***Nasdaq****. The market includes electronic trading, broker-dealer platforms, and alternative trading systems (ATS). Liquidity is often provided by market makers and HFTs.*

**Definition 4** ((1.4) Bid, Ask, and Spread)**.** *The* ***ask price*** *is the lowest price a seller is willing to accept; the* ***bid price*** *is the highest a buyer is willing to pay. The* ***bid-ask spread*** *is defined as*

$$\textit{Spread} = \textit{Best Ask} - \textit{Best Bid} > 0.$$

*For immediate execution: buyers pay the ask; sellers receive the bid.*

**Definition 5** ((1.5) Financial Derivative)**.** *A* ***financial derivative*** *is a financial instrument whose value depends on (or is derived from) an underlying asset. Examples: stocks, indices, interest rates, commodities, currencies, credit instruments.*

**Definition 6** ((1.6) Forward Contract)**.** *A* ***forward contract*** *is an agreement between two parties to buy or sell an asset at a fixed* ***forward price*** *on a specified* ***maturity date****. The buyer holds the* ***long position****; the seller holds the* ***short position****.*

**Proposition 3** ((1.C) Forward Contract Mechanics)**.** *The spot price $S_t$ denotes the current market price at time $t$. At maturity $T$, the contract payoff is:*

$$\textit{Long: } S_T - F, \quad \textit{Short: } F - S_T$$

*Instead of physical delivery, parties may exchange this payoff in cash.*

**Definition 7** ((1.7) Payoff Function)**.** *A* ***payoff function*** *represents the mark-to-market value of a position at maturity in terms of the underlying asset's market price. Payoffs may be positive or negative, and positions may be* ***long*** *(own/buy) or* ***short*** *(sell/owe).*

**Definition 8** ((1.11) Call and Put Options)**.** *A* ***call option*** *gives the buyer the right (not obligation) to buy an asset at price $K$ before or on date $T$. A* ***put option*** *gives the buyer the right to sell. $K$ is the strike price; $T$ is the expiration date.* ***European options*** *can only be exercised at $T$;* ***American options*** *at any time up to $T$.*

**Proposition 4** ((1.D) Options vs Forwards/Futures)**.** *Forwards and futures are* ***obligations*** *to trade, typically entered at zero cost. Options provide a* ***right****, not obligation, and thus require the buyer to pay a* ***premium****.*

**Proposition 5** ((1.E) Derivatives Markets and Regulation)**.** *Derivatives may be traded on exchanges (standardized contracts, centrally cleared) or over-the-counter (customized, bilateral risk). In the U.S., futures and swaps are regulated by the* ***CFTC****, especially post–2008 under Dodd-Frank reforms. OTC contracts often use ISDA Master Agreements.*

**Proposition 6** ((1.F) Derivatives Market Participants)**.** *Three categories of users:*

- ***Speculators*** *trade based on market expectations*
- ***Hedgers*** *reduce exposure to risk*
- ***Arbitrageurs*** *exploit price discrepancies across markets*

# Math 174E Lecture 4–5: Options, Hedging, and Arbitrage

**Definition 9** ((1.14) Net Profit)**.** *The* ***net profit*** *(or net P&L) of a financial position is the difference between the payoff at maturity and the initial setup cost (e.g., premium).*

*At maturity $T$, for a European option:*

$$\begin{aligned} \textit{Call (long):} \quad & \max(S_T - K, 0) - C_0(K,T) \\ \textit{Put (long):} \quad & \max(K - S_T, 0) - P_0(K,T) \\ \textit{Call (short):} \quad & C_0(K,T) - \max(S_T - K, 0) \\ \textit{Put (short):} \quad & P_0(K,T) - \max(K - S_T, 0) \end{aligned}$$

*The break-even stock price at maturity is:*

$$\textit{Call: } S_T = K + C_0(K,T) \quad \textit{Put: } S_T = K - P_0(K,T)$$

**Proposition 7** ((4.A) Option Position Payoffs))**.** *At maturity $T$, payoffs are given by:*

$$\begin{aligned} \textit{Call (long):} \quad & (S_T - K)^+ \\ \textit{Call (short):} \quad & -(S_T - K)^+ \\ \textit{Put (long):} \quad & (K - S_T)^+ \\ \textit{Put (short):} \quad & -(K - S_T)^+ \end{aligned}$$

*with notation $x^+ = \max(x, 0)$.*

**Proposition 8** ((4.B) Option Quote Structure))**.** *Options are traded on exchanges like CBOE. Quotes include bid and ask prices for each strike and maturity. One option contract typically corresponds to 100 shares. Expiration is usually the third Friday of the month.*

**Proposition 9** ((4.C) Hedging with Derivatives))**.** *A forward contract locks in the price at which a company will sell or buy an asset in the future, protecting against adverse price movements. An option contract provides downside protection (insurance) while still allowing participation in favorable price changes. The hedger pays a premium for this asymmetric risk profile.*

**Proposition 10** ((4.D) Speculation Using Options))**.** *Options allow speculative positions with limited downside (the premium paid) and amplified upside. Buying a call is bullish; buying a put is bearish. Options provide leverage, since a small premium offers exposure to large price moves.*

**Proposition 11** ((4.E) Option Notation))**.** *Let $K$ be the strike price and $T$ the maturity. The premium at time 0 is denoted*

$$C_0(K,T) \textit{ for a call,} \quad P_0(K,T) \textit{ for a put}$$

*Let $(S_t)_{0 \le t \le T}$ be the spot price process of the underlying.*

**Definition 10** ((1.21) Arbitrage Opportunity)**.** *An* ***arbitrage opportunity*** *is a trading strategy with the following properties:*

1. *No initial external capital is required.*

2. *The strategy guarantees a non-negative payoff, which is strictly positive with strictly positive probability.*

*Such opportunities imply mispricing and disappear once exploited. Arbitrage enforces pricing consistency in financial markets.*

**Proposition 12** ((5.A) Arbitrage-Free Pricing Principle)). *The prices of financial derivatives are constrained by the* ***no-arbitrage principle****: there should be no strategy with zero cost and guaranteed profit. This principle underlies the valuation of forwards, futures, and options. In particular:*

- *Forward prices must be set to eliminate arbitrage between spot and forward markets.*

- *Option premiums must not allow guaranteed profits against the underlying asset.*

**Proposition 13** ((5.B) Mispriced Call Option Example)). *If the call premium equals the spot price, $C_0(K,T) = S_0$, an arbitrage is possible:*

$$\text{At } t = 0: \quad \text{Sell call (receive } C_0), \quad \text{Buy stock (cost } S_0)$$

$$\text{Net initial cost } = 0$$

$$\text{At } t = T: \begin{cases} S_T > K: & \text{Call is exercised, receive } K, \text{ net gain } K \\ S_T < K: & \text{Call not exercised, sell stock at } S_T \end{cases}$$

*Since $\min(S_T, K) > 0$, this violates the no-arbitrage principle. Hence, $C_0 < S_0$ must hold.*

**Proposition 14** ((5.C) Arbitrage Enforcement)). *The existence of arbitrageurs ensures pricing stays consistent. Arbitrage opportunities rarely persist due to low transaction costs for institutional players. This makes arbitrage a fundamental market force — like gravity in physics — for ensuring fair pricing.*

**Proposition 15** ((6.A) Put-Call Parity for European Options)). *Let $C_0(K,T)$ be the premium of a European call and $P_0(K,T)$ that of a European put with the same strike $K$ and maturity $T$. Assume no dividends and a constant risk-free rate $r$. Then the following relation holds:*

$$C_0(K,T) - P_0(K,T) = S_0 - Ke^{-rT}$$

*This identity is known as* ***put-call parity*** *and ensures no arbitrage between options and the underlying asset.*

**Proposition 16** ((6.B) Arbitrage from Violations of Put-Call Parity)). *If put-call parity is violated, an arbitrage opportunity exists. For example, if*

$$C_0(K,T) - P_0(K,T) > S_0 - Ke^{-rT},$$

*then an arbitrageur can:*

- *Sell the call and buy the put (net inflow)*

- *Buy the stock and borrow $Ke^{-rT}$ at rate $r$*

*At maturity, all positions net to a profit regardless of $S_T$. Thus, prices must adjust to restore parity.*

# Math 174E Lecture 6: Futures Contracts and Margins

**Definition 11** ((2.1) Futures Contract). *A* ***futures contract*** *is a standardized agreement traded on an exchange to buy or sell an asset at a predetermined price on a specified future date. The contract specifies the underlying asset, contract size, delivery details, and tick size.*

**Proposition 17** ((2.A) Futures Market Structure)). *Futures are actively traded globally on exchanges like the CME Group and Eurex. Contracts exist on commodities, interest rates, currencies, stock indices, and more. Trading occurs electronically via a limit order book, and prices reflect supply and demand under no-arbitrage constraints.*

**Proposition 18** ((2.B) Key Features of Futures Contracts)**.** *Each futures contract includes:*

- *Underlying asset and contract size*
- *Delivery month and location*
- *Price quote format and tick size*
- *Position limits and price movement limits*

*Contracts are* ***marked to market daily****, and gains/losses are realized through margin accounts.*

**Definition 12** ((2.2) Margin Account and Daily Settlement)**.** *A* ***margin account*** *is a deposit required to manage counterparty risk. There are three types:*

- ***Initial margin****: amount deposited at trade initiation*
- ***Maintenance margin****: minimum required to avoid a margin call*
- ***Variation margin****: additional funds required to restore margin balance*

*Daily settlement reflects gains or losses using the current futures price (settlement price).*

**Proposition 19** ((2.C) Closing a Futures Position)**.** *A futures position may be closed prior to maturity by taking an opposite position. The profit or loss is determined by:*

$$\textit{Long: } F_{t_2} - F_{t_1} \quad \textit{Short: } F_{t_1} - F_{t_2}$$

*Gains and losses are accounted for daily through marking-to-market.*

**Proposition 20** ((2.D) Delivery and Settlement)**.** *Most futures contracts are closed before delivery. If not:*

- ***Physical delivery****: the short issues a notice to deliver the asset*
- ***Cash settlement****: used when delivery is impractical (e.g., S&P 500 futures)*

*Final settlement price is the spot price at the closing date.*

**Proposition 21** ((2.E) Properties of Futures Prices)**.**

- ***Convergence****: as maturity approaches, futures price converges to the spot price.*
- ***Contango****: futures price increases with maturity.*
- ***Backwardation****: futures price decreases with maturity.*

*Deviations between futures and spot prices can lead to arbitrage opportunities.*

**Proposition 22** ((2.F) Forward vs. Futures Contracts)**.**

| ***Feature*** | ***Forward Contract*** | ***Futures Contract*** |
|---|---|---|
| *Trading Venue* | *Private* | *Exchange* |
| *Standardization* | *Custom* | *Standardized* |
| *Settlement* | *At maturity* | *Daily (mark-to-market)* |
| *Delivery* | *Specified date* | *Range or closed early* |
| *Credit Risk* | *Present* | *Minimal* |

# Math 174E Lecture 7–8: Hedging with Futures

**Definition 13** ((3.1) Hedging)**.** *A* ***hedge*** *is an investment made with the goal of reducing or eliminating the risk of adverse price movements in another asset. Hedging often involves derivatives and aims to match cash flows to neutralize financial exposure.*

**Proposition 23** ((3.A) Static Hedging with Futures)**.** *Hedging with futures involves taking a futures position to lock in prices. Two primary types:*

- ***Short hedge***: *used when selling an asset in the future — take a short futures position.*
- ***Long hedge***: *used when buying an asset in the future — take a long futures position.*

*Futures positions are typically closed before maturity (no delivery).*

**Definition 14** ((3.2) Basis and Basis Risk)**.** ***Basis*** *is defined as the difference between the spot price and futures price:*

$$Basis = S_t - F_t$$

***Basis risk*** *is the risk that the basis changes unpredictably over time, especially if the asset to be hedged differs from the asset underlying the futures contract.*

**Proposition 24** ((3.B) Hedge Pricing Formula)**.** *Let $t_1$ be the time the hedge is initiated and $t_2$ the time it is closed. Then:*

- ***Short hedge effective sale price***: $F_{t_1} + b_{t_2}$
- ***Long hedge effective purchase price***: $F_{t_1} + b_{t_2}$

*The uncertainty of $b_{t_2}$ is the source of basis risk.*

**Proposition 25** ((3.C) Contract Selection in Hedging)**.** *To select a futures contract:*

1. *Choose a delivery month just after the hedging horizon.*
2. *If no contract exists on the asset, use one with a highly correlated asset — this is called* ***cross hedging****.*

**Definition 15** ((3.3) Hedge Ratio)**.** *The* ***hedge ratio*** *is defined as:*

$$h = \frac{N_F}{N_A}$$

*where $N_F$ is the size of the futures position and $N_A$ is the size of the position being hedged. For perfect hedges with identical assets, $h = 1$.*

**Proposition 26** ((3.D) Minimum Variance Hedge Ratio)**.** *To minimize the variance of the hedged position, the optimal hedge ratio is:*

$$h^* = \rho \cdot \frac{\sigma_S}{\sigma_F}$$

*where $\rho$ is the correlation between spot and futures price changes, and $\sigma_S, \sigma_F$ are their respective standard deviations. The optimal number of contracts is:*

$$N^* = \frac{h^* \cdot N_A}{Q_F}$$

*where $Q_F$ is the size of one futures contract.*

**Proposition 27** ((3.E) Interpretation of Minimum Variance Hedge Ratio)**.** *The optimal hedge ratio $h^*$ is the slope of the regression line when regressing spot price changes on futures price changes. It represents the best linear predictor of spot price changes using futures price changes.*

**Proposition 28** ((3.F) Example: Airline Fuel Hedge)**.** *An airline hedges a future purchase of 2 million gallons of jet fuel using heating oil futures. Given:*

$$\sigma_F = 0.0313, \quad \sigma_S = 0.0263, \quad \rho = 0.928 \Rightarrow h^* = 0.78$$

*If each contract covers 42,000 gallons:*

$$N^* = \frac{0.78 \cdot 2{,}000{,}000}{42{,}000} \approx 37 \text{ contracts}$$

# Math 174E Lecture 9: Introduction to Interest Rates

**Definition 16** ((4.1) Interest Rate)**.** *An* ***interest rate*** *quantifies the amount of compensation a borrower must pay a lender for the use of money over time. It reflects the* ***time value of money*** *and often incorporates credit risk.*

**Proposition 29** ((4.A) Units and Quoting of Interest Rates))**.** *Interest rates are often expressed in* ***basis points****:*

$$\text{1 basis point} = 0.01\% = 0.0001$$

*Rates differ by context and borrower risk profile. Examples include deposit rates, prime borrowing rates, and mortgage rates.*

**Proposition 30** ((4.B) Types of Interest Rates))**.**

- ***Treasury Rates****: Yields on government-issued bills and bonds; considered* ***risk-free****.*
- ***Overnight Rates****: Short-term rates such as the U.S. federal funds rate.*
- ***Repo Rates****: Secured borrowing rates under repurchase agreements. Include overnight and term repos.*

**Definition 17** ((4.2) Reference Rates))**.**

- ***LIBOR*** *(London Interbank Offered Rate): Formerly the benchmark for interbank lending and derivatives. Phased out after 2021 due to manipulation concerns.*
- ***SOFR*** *(Secured Overnight Financing Rate): A replacement for LIBOR in U.S. markets. Based on overnight repo transactions and considered a* ***risk-free rate****.*

*SOFR may be adjusted with a spread to reflect term and credit conditions.*